Module 6: Two Dimensional Problems in Polar Coordinate System

6.3.1 BARS WITH LARGE INITIAL CURVATURE

There are practical cases of bars, such as hooks, links and rings, etc. which have large initial curvature. In such a case, the dimensions of the cross-section are not very small in comparison with either the radius of curvature or with the length of the bar. The treatment that follows is based on the theory due to Winkler and Bach.

6.3.2 WINKLER'S – BACH THEORY

Assumptions

1. Transverse sections which are plane before bending remain plane even after bending.
2. Longitudinal fibres of the bar, parallel to the central axis exert no pressure on each other.
3. All cross-sections possess a vertical axis of symmetry lying in the plane of the centroidal axis passing through *C* (Figure 6.11)
4. The beam is subjected to end couples *M*. The bending moment vector is normal throughout the plane of symmetry of the beam.

Winkler-Bach Formula to Determine Bending Stress or Normal Stress (Also known as Circumferential Stress)

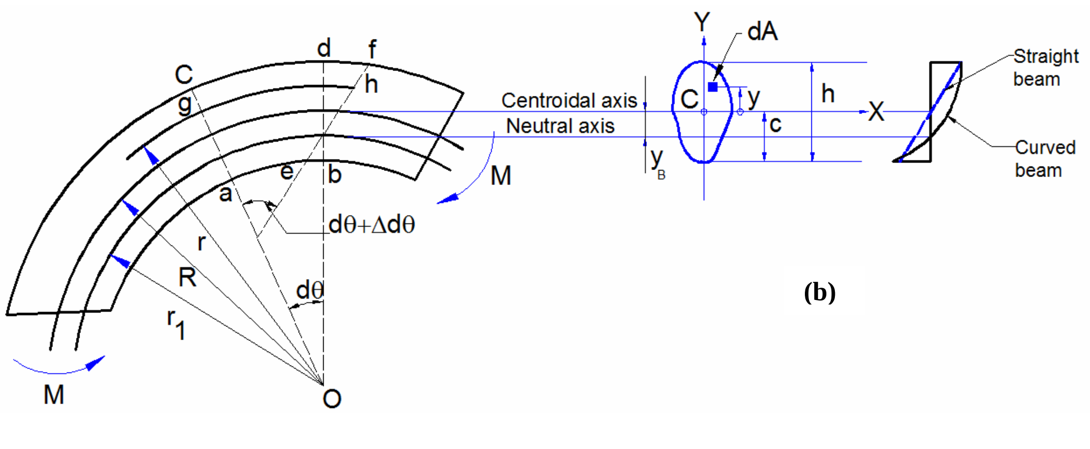


Figure 6.11 Beam with large initial curvature

Consider a curved beam of constant cross-section, subjected to pure bending produced by couples M applied at the ends. On the basis of plane sections remaining plane, we can state that the total deformation of a beam fiber obeys a linear law, as the beam element rotates through small angle $\Delta d\theta$. But the tangential strain ε_θ does not follow a linear relationship.

The deformation of an arbitrary fiber, $gh = \varepsilon_c R d\theta + y\Delta d\theta$

where ε_c denotes the strain of the centroidal fiber

But the original length of the fiber $gh = (R + y)\, d\theta$

Therefore, the tangential strain in the fiber $gh = \varepsilon_\theta = \dfrac{[\varepsilon_c R d\theta + y\Delta d\theta]}{(R+y)d\theta}$

Using Hooke's Law, the tangential stress acting on area dA is given by

$$\sigma_\theta = \frac{\varepsilon_c R + y(\Delta d\theta / d\theta)}{(R+y)} E \qquad (6.61)$$

Let angular strain $\dfrac{\Delta d\theta}{d\theta} = \lambda$

Hence, Equation (6.61) becomes

$$\sigma_\theta = \frac{\varepsilon_c R + y\lambda}{(R+y)} E \qquad (6.62)$$

Adding and subtracting $\varepsilon_c y$ in the numerator of Equation (6.62), we get,

$$\sigma_\theta = \frac{\varepsilon_c R + y\lambda + \varepsilon_c y - \varepsilon_c y}{(R+y)} E$$

Simplifying, we get

$$\sigma_\theta = \left[\varepsilon_c + (\lambda - \varepsilon_c)\frac{y}{(R+y)}\right] E \qquad (6.63a)$$

The beam section must satisfy the conditions of static equilibrium,

$F_z = 0$ and $M_x = 0$, respectively:

$$\therefore \int \sigma_\theta dA = 0 \quad \text{and} \quad \int \sigma_\theta y dA = M \qquad (6.63b)$$

Substituting the above boundary conditions (6.63b) in (6.63a), we get

$$0 = \int\left[\varepsilon_c + (\lambda - \varepsilon_c)\frac{y}{(R+y)}\right]dA$$

or $\int \varepsilon_c dA = -(\lambda - \varepsilon_c)\int \frac{y}{(R+y)}dA$

or $\varepsilon_c \int dA = -(\lambda - \varepsilon_c)\int \frac{y}{(R+y)}dA$ (6.63c)

Also,

M = $\left[\varepsilon_c \int ydA + (\lambda - \varepsilon_c)\int \frac{y^2}{(R+y)}dA\right]E$ (6.63d)

Here $\int dA = A$, and since y is measured from the centroidal axis, $\int ydA = 0$.

Let $\int \frac{y}{(R+y)}dA = -mA$

Or $m = -\frac{1}{A}\int \frac{y}{(R+y)}dA$

Therefore, $\int \frac{y^2}{(R+y)}dA = \int\left(y - \frac{Ry}{(R+y)}\right)dA$

$= \int ydA - \int \frac{Ry}{(R+y)}dA$

$= 0 - R\,[-mA]$

$\therefore \quad \int \frac{y^2}{(R+y)}dA = mRA$

Substituting the above values in (6.63c) and (6.63d), we get,

$\varepsilon_c = (\lambda - \varepsilon_c)m$

and $M = E\,(\lambda - \varepsilon_c)\,mAR$

From the above, we get

$\varepsilon_c = \frac{M}{AER}$ and $\lambda = \frac{1}{AE}\left(\frac{M}{R} + \frac{M}{mR}\right)$ (6.63e)

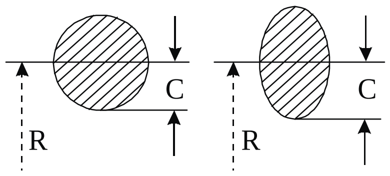
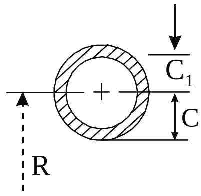
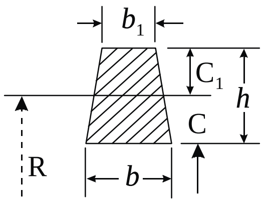
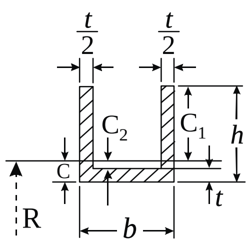
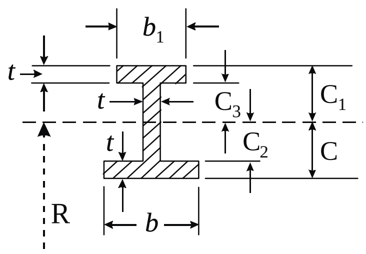
Substitution of the values of Equation (6.63e) into Equation (6.63a) gives an expression for the tangential stress in a curved beam subject to pure bending.

Therefore, $\sigma_\theta = \frac{M}{AR}\left[1+\frac{y}{m(R+y)}\right]$ (6.64)

The above expression for σ_θ is generally known as the "Winkler-Bach formula". The distribution of stress σ_θ is given by the hyperbolic (and not linear as in the case of straight beams) as shown in the Figure 6.11 (b).

In the above expression, the quantity *m* is a pure number, and is the property of each particular shape of the cross-section. Table 6.1 gives the formula for *m* for various shapes of the cross-section.

Table 6.1. Value m for various shapes of cross-section

Cross-section	Formula for '*m*'
A 	$m = -1 + 2\left(\frac{R}{C}\right)^2 - 2\left(\frac{R}{C}\right)\sqrt{\left(\frac{R}{C}\right)^2 - 1}$
B 	$m = -1 + \frac{2R}{C^2 - C_1^2}\left[\sqrt{R^2 - C_1^2} - \sqrt{R^2 - C^2}\right]$
C 	$m = -1 + R/Ah\ \{[b_1h + (R + C_1)(b - b_1)]$ $1n\left(\frac{R + C_1}{R - C}\right) - (b - b_1)h\}$ For Rectangular Section: $C = C_1$; $b = b_1$ For Triangular Section: $b_1 = 0$
D 	$m = -1 + \frac{R}{A}[t.1n\ (R + C_1) + (b - t).1n$ $(R - C_2) - b.1n\ (R - C)]$
E 	$m = -1 + \frac{R}{A}[b_1.1n\ (R + C_1) + (t - b_1).1n$ $(R + C_3) + (b - t).1n\ (R - C_2) - b.1n\ (R - C)]$

Sign Convention

The following sign convention will be followed:

1. A bending moment *M* will be taken as positive when it is directed towards the concave side of the beam (or it decreases the radius of curvature), and negative if it increases the radius of curvature.

2. '*y*' is positive when measured towards the convex side of the beam, and negative when measured towards the concave side (or towards the centre of curvature).

3. With the above sign convention, if σ_θ is positive, it denotes tensile stress while negative sign means compressive stress.

The distance between the centroidal axis ($y = 0$) and the neutral axis is found by setting the tangential stress to zero in Equation (5.15)

$$\therefore \quad 0 = \frac{M}{AR}\left[1 + \frac{y}{m(R+y)}\right]$$

$$\text{or} \quad 1 = -\frac{y_n}{m(R-y_n)}$$

where y_n denotes the distance between axes as indicated in Figure 5.2. From the above,

$$y_n = -\frac{mR}{(m+1)}$$

This expression is valid for pure bending only.

However, when the beam is acted upon by a normal load *P* acting through the centriod of cross-sectional area *A*, the tangential stress given by Equation (5.15) is added to the stress produced by this normal load *P*. Therefore, for this simple case of superposition, we have

$$\sigma_\theta = \frac{P}{A} + \frac{M}{AR}\left[1 + \frac{y}{m(R+y)}\right] \tag{6.65}$$

As before, a negative sign is associated with a compressive load *P*.

6.3.3 STRESSES IN CLOSED RINGS

Crane hook, split rings are the curved beams that are unstrained at one end or both ends. For such beams, the bending moment at any section can be calculated by applying the equations of statics directly. But for the beams having restrained or fixed ends such as a close ring, equations of equilibrium are not sufficient to obtain the solution, as these beams are statically indeterminate. In such beams, elastic behaviour of the beam is considered and an additional condition by considering the deformation of the member under given load is developed as in the case of statically indeterminate straight beam.

Now, consider a closed ring shown in figure 6.12 (a), which is subjected to a concentrated load *P* along a vertical diametrical plane.

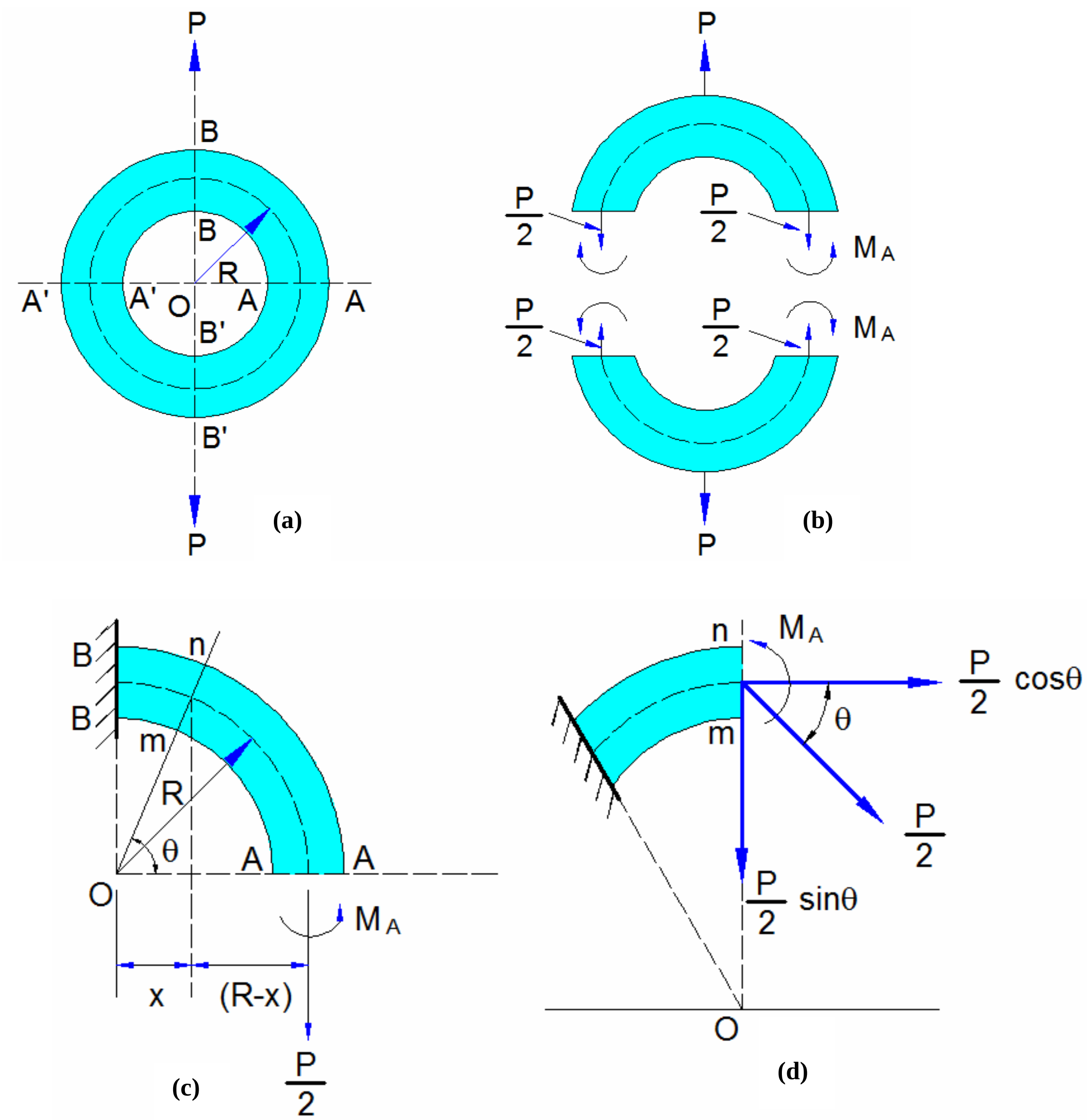


Figure 6.12 Closed ring subjected to loads

The distribution of stress in upper half of the ring will be same as that in the lower half due to the symmetry of the ring. Also, the stress distribution in any one quadrant will be same as in another. However, for the purposes of analysis, let us consider a quadrant of the circular ring as shown in the Figure 6.12 (c), which may be considered to be fixed at the section BB and at section AA subjected to an axial load $\frac{P}{2}$ and bending moment M_A. Here the magnitude and the sign of the moment M_A are unknown.

Now, taking the moments of the forces that lie to the one side of the section, then we get,

$$M_{mn} = -M_A + \frac{P}{2}(R - x)$$

But from Figure, $x = R\cos\theta$

$$\therefore M_{mn} = -M_A + \frac{P}{2}(R - R\cos\theta)$$

$$\therefore M_{mn} = -M_A + \frac{PR}{2}(1 - \cos\theta) \quad \text{(a)}$$

The moment M_{mn} at the section MN cannot be determined unless the magnitude of M_A is known. Resolving $\frac{P}{2}$ into normal and tangential components, we get

Normal Component, producing uniform tensile stress = $N = \frac{1}{2}P\cos\theta$

Tangential component, producing shearing stress = $T = \frac{1}{2}P\sin\theta$

Determination of M_A

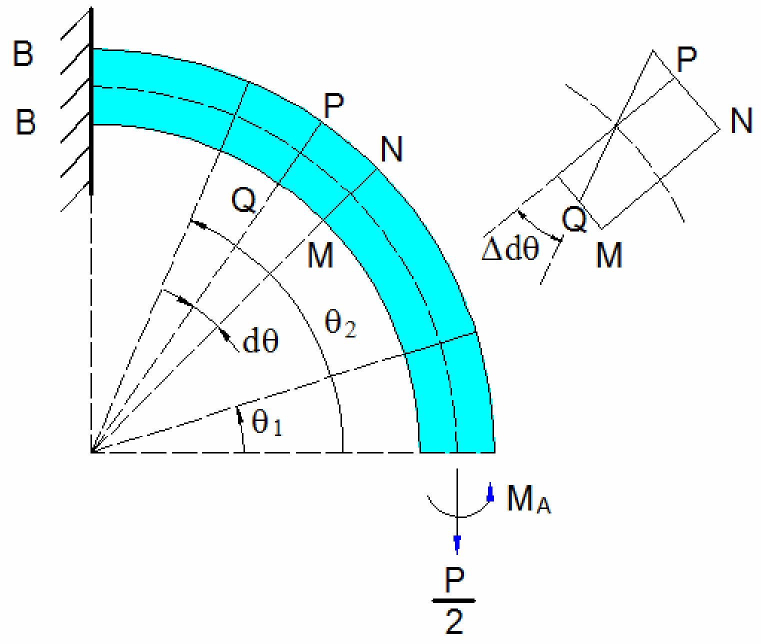


Figure 6.13 Section PQMN

Consider the elastic behavior of the two normal sections MN and PQ, a differential distance apart. Let the initial angle $d\theta$ between the planes of these two sections change by an amount $\Delta d\theta$ when loads are applied.

Therefore, the angular strain = $\omega = \dfrac{\Delta d\theta}{d\theta}$

i.e., $\Delta d\theta = \omega \, d\theta.$

Therefore, if we are interested in finding the total change in angle between the sections, that makes an angle θ_1 and θ_2 with the section AA, the expression $\int_{\theta_2}^{\theta_1} \omega \, d\theta$ will give that angle.

But in the case of a ring, sections AA and BB remain at right angles to each other before and after loading. Thus, the change in the angle between these planes is equal to zero. Hence

$$\int_o^{\pi/2} \omega \, d\theta = 0 \qquad \text{(b)}$$

In straight beams the rate of change of slope of the elastic curve is given by $\frac{d^2y}{dx^2} = \frac{M}{EI}$.

Whereas in initially curved beam, the rate of change of slope of the elastic curve is $\frac{\Delta d\theta}{Rd\theta}$, which is the angle change per unit of arc length.

Now, $\frac{\Delta d\theta}{Rd\theta} = \frac{\omega}{R} = \frac{M}{EI} = \frac{M_{mn}}{EI}$ for curved beams

Or $w = \frac{R M_{mn}}{EI}$

Substituting the above in equation (b), we get

$$\int_o^{\frac{\pi}{2}} \frac{R.M_{mn}}{EI} d\theta = 0$$

since R, E and I are constants,

$$\therefore \int_o^{\frac{\pi}{2}} M_{mn} d\theta = 0$$

From Equation (a), substituting the value of M_{mn}, we obtain

$$-\int_0^{\frac{\pi}{2}} M_A\, d\theta + \frac{1}{2}PR\int_0^{\frac{\pi}{2}} d\theta - \frac{1}{2}PR\int_0^{\frac{\pi}{2}} \cos\theta\, d\theta = 0$$

Integrating, we get

$$-M_A[\theta]_0^{\frac{\pi}{2}} + \frac{1}{2}PR[\theta]_0^{\frac{\pi}{2}} - \frac{1}{2}PR[\sin\theta]_0^{\frac{\pi}{2}} = 0$$

$$-M_A\left(\frac{\pi}{2}\right) + \frac{1}{2}PR\left(\frac{\pi}{2}\right) - \frac{1}{2}PR\left(\sin\frac{\pi}{2}\right) = 0$$

Thus $M_A = \frac{PR}{2}\left(1 - \frac{2}{\pi}\right)$

Therefore, knowing M_A, the moment at any section such as MN can be computed and then the normal stress can be calculated by curved beam formula at any desired section.

6.3.4 NUMERICAL EXAMPLES

Example 6.1
Given the following stress function

$$\phi = \frac{P}{\pi} r\theta \cos\theta$$

Determine the stress components σ_r, σ_θ *and* $\tau_{r\theta}$

***Solution*:** The stress components, by definition of ϕ, are given as follows

$$\sigma_r = \left(\frac{1}{r}\right)\frac{\partial\phi}{\partial r} + \left(\frac{1}{r^2}\right)\frac{\partial^2\phi}{\partial\theta^2} \qquad \text{(i)}$$

$$\sigma_\theta = \frac{\partial^2\phi}{\partial r^2} \qquad \text{(ii)}$$

$$\tau_{r\theta} = \left(\frac{1}{r^2}\right)\frac{\partial\phi}{\partial\theta} - \left(\frac{1}{r}\right)\frac{\partial^2\phi}{\partial r\partial\theta} \qquad \text{(iii)}$$

The various derivatives are as follows:

$$\frac{\partial\phi}{\partial r} = \frac{P}{\pi}\theta\cos\theta$$

$$\frac{\partial^2\phi}{\partial r^2} = 0$$

$$\frac{\partial\phi}{\partial\theta} = \frac{P}{\pi} r(-\theta\sin\theta + \cos\theta)$$

$$\frac{\partial^2\phi}{\partial\theta^2} = -\frac{P}{\pi} r(\theta\cos\theta + 2\sin\theta)$$

$$\frac{\partial^2\phi}{\partial r\partial\theta} = \frac{P}{\pi}(-\theta\sin\theta + \cos\theta)$$

Substituting the above values in equations (i), (ii) and (iii), we get

$$\sigma_r = \left(\frac{1}{r}\right)\frac{P}{\pi}\theta\cos\theta - \left(\frac{1}{r^2}\right)\frac{P}{\pi} r(\theta\cos\theta + 2\sin\theta)$$

$$= \left(\frac{1}{r}\right)\frac{P}{\pi}\theta\cos\theta - \left(\frac{1}{r}\right)\frac{P}{\pi}\theta\cos\theta - \left(\frac{1}{r}\right)\frac{P}{\pi}2\sin\theta$$

$$\therefore \sigma_r = -\frac{2}{r}\frac{P}{\pi}\sin\theta$$

$$\sigma_\theta = \frac{\partial^2\phi}{\partial r^2} = 0$$

$$\tau_{r\theta} = \left(\frac{1}{r^2}\right)\frac{P}{\pi} r(-\theta\sin\theta + \cos\theta) - \left(\frac{1}{r}\right)\frac{P}{\pi}(-\theta\sin\theta + \cos\theta)$$

$$\therefore \tau_{r\theta} = 0$$

Therefore, the stress components are

$$\sigma_r = -\left(\frac{2}{r}\right)\frac{P}{\pi}\sin\theta$$

$$\sigma_\theta = 0$$

$$\tau_{r\theta} = 0$$

Example 6.2

A thick cylinder of inner radius 10*cm* and outer radius 15*cm* is subjected to an internal pressure of 12*MPa*. Determine the radial and hoop stresses in the cylinder at the inner and outer surfaces.

***Solution*:** The radial stress in the cylinder is given by

$$\sigma_r = \left(\frac{p_i a^2 - p_o b^2}{b^2 - a^2}\right) - \left(\frac{p_i - p_o}{b^2 - a^2}\right)\frac{a^2 b^2}{r^2}$$

The hoop stress in the cylinder is given by

$$\sigma_\theta = \left(\frac{p_i a^2 - p_o b^2}{b^2 - a^2}\right) + \left(\frac{p_i - p_o}{b^2 - a^2}\right)\frac{a^2 b^2}{r^2}$$

As the cylinder is subjected to internal pressure only, the above expressions reduce to

$$\sigma_r = \left(\frac{p_i a^2}{b^2 - a^2}\right) - \left(\frac{p_i}{b^2 - a^2}\right)\frac{a^2 b^2}{r^2}$$

and $\sigma_\theta = \left(\frac{p_i a^2}{b^2 - a^2}\right) + \left(\frac{p_i}{b^2 - a^2}\right)\frac{a^2 b^2}{r^2}$

Stresses at inner face of the cylinder (i.e., at r = 10 cm):

Radial stress = $\sigma_r = \left[\frac{12 \times (0.1)^2}{(0.15)^2 - (0.1)^2}\right] - \left[\frac{(0.15)^2 (0.1)^2}{(0.1)^2}\right]\left[\frac{12}{(0.15)^2 - (0.1)^2}\right]$

= 9.6 – 21.6

or σ_r = -12 *MPa*

Hoop stress = σ_θ = $\left[\frac{12\times(0.1)^2}{(0.15)^2-(0.1)^2}\right]+\left[\frac{12}{(0.15)^2-(0.1)^2}\right]\left[\frac{(0.15)^2(0.1)^2}{(0.1)^2}\right]$

= 9.6 + 21.6

or σ_θ = 31.2 *MPa*

Stresses at outerface of the cylinder (i.e., at r = 15 cm):

Radial stress = σ_r = $\left[\frac{12\times(0.1)^2}{(0.15)^2-(0.1)^2}\right]-\left[\frac{12}{(0.15)^2-(0.1)^2}\right]\left[\frac{(0.1)^2(0.15)^2}{(0.15)^2}\right]$

σ_r = 0

Hoop stress = σ_θ = $\left[\frac{12\times(0.1)^2}{(0.15)^2-(0.1)^2}\right]+\left[\frac{(0.1)^2(0.15)^2}{(0.15)^2}\right]\left[\frac{12}{(0.15)^2-(0.1)^2}\right]$

= 9.6 + 9.6

or σ_θ = 19.2 *MPa*

Example 6.3

A steel tube, which has an outside diameter of 10cm and inside diameter of 5cm, is subjected to an internal pressure of 14 *MPa* and an external pressure of 5.5 *MPa*. Calculate the maximum hoop stress in the tube.

***Solution*:** The maximum hoop stress occurs at $r = a$.

Therefore, Maximum hoop stress = $(\sigma_\theta)_{max}$ = $\left[\frac{p_i a^2 - p_0 b^2}{b^2-a^2}\right]+\left[\frac{p_i - p_0}{b^2-a^2}\right]\left[\frac{a^2 b^2}{a^2}\right]$

$$= \left[\frac{p_i a^2 - p_0 b^2}{b^2-a^2}\right]+\left[\frac{p_i - p_0}{b^2-a^2}\right]b^2$$

$$= \frac{p_i a^2 - p_0 b^2 + p_i b^2 - p_0 b^2}{b^2-a^2}$$

$$(\sigma_\theta)_{max} = \frac{p_i(a^2+b^2)-2p_0 b^2}{b^2-a^2}$$

Therefore, $(\sigma_\theta)_{max} = \frac{14[(0.05)^2+(0.1)^2]-2\times5.5\times(0.1)^2}{(0.1)^2-(0.05)^2}$

Or $(\sigma_\theta)_{max}$ = 8.67 *MPa*

Example 6.4

A steel cylinder which has an inside diameter of 1m is subjected to an internal pressure of 8 *MPa*. Calculate the wall thickness if the maximum shearing stress is not to exceed 35 *MPa*.

***Solution*:** The critical point lies on the inner surface of the cylinder, i.e., at $r = a$.

We have, Radial stress = $\sigma_r = \left[\frac{p_i a^2 - p_0 b^2}{b^2 - a^2}\right] - \left[\frac{p_i - p_0}{b^2 - a^2}\right]\frac{a^2 b^2}{r^2}$

At $r = a$ and $p_0 = 0$,

$$\sigma_r = \left[\frac{p_i a^2 - 0}{b^2 - a^2}\right] - \left[\frac{p_i - 0}{b^2 - a^2}\right]\frac{a^2 b^2}{a^2}$$

$$= \frac{p_i a^2 - p_i b^2}{b^2 - a^2}$$

$$= \frac{-p_i (b^2 - a^2)}{(b^2 - a^2)}$$

Therefore, $\sigma_r = -p_i$

Similarly,

Hoop stress = $\sigma_\theta = \left[\frac{p_i a^2 - p_0 b^2}{b^2 - a^2}\right] + \left[\frac{p_i - p_0}{b^2 - a^2}\right]\frac{a^2 b^2}{r^2}$

At $r = a$ and $p_0 = 0$,

$$\sigma_\theta = \left[\frac{p_i a^2 - 0}{b^2 - a^2}\right] + \left[\frac{p_i - 0}{b^2 - a^2}\right]\frac{a^2 b^2}{a^2}$$

$$\sigma_\theta = \frac{p_i (a^2 + b^2)}{(b^2 - a^2)}$$

Here the maximum and minimum stresses are

$\sigma_3 = -p_i$ and $\sigma_1 = \sigma_\theta$

But the maximum shear stress = $\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_3)$

i.e. $\tau_{max} = \frac{1}{2}\left[\frac{p_i (a^2 + b^2)}{(b^2 - a^2)} + p_i\right]$

$$= \frac{1}{2}\left[\frac{p_i a^2 + p_i b^2 + p_i b^2 - p_i a^2}{(b^2 - a^2)}\right]$$

$$35 = \frac{p_i b^2}{(b^2 - a^2)}$$

i.e., $35 = \dfrac{8 \times b^2}{(b^2 - a^2)}$

$35b^2 - 35a^2 = 8b^2$

$35b^2 - 8b^2 = 35a^2$

$35b^2 - 8b^2 = 35(0.5)^2$

Therefore, $b = 0.5693$

If t is the thickness of the cylinder, then

$b = 0.5 + t = 0.5693$

$\therefore t = 0.0693\ m$ or $69.3\ mm$.

Example 6.5

The circular link shown in Figure 6.14 has a circular cross-section 3cm in diameter. The inside diameter of the ring is 4cm. The load *P* is 1000 kg. Calculate the stress at *A* and *B*. Compare the values with those found by the straight beam formula. Assume that the material is not stressed above its elastic strength.

***Solution*:**

Cross-sectional area = $A = \dfrac{\pi}{4}(3)^2 = 7.06\ cm^2$.

For circular cross-section m is given by

$$m = -1+2\left(\frac{R}{c}\right)^2 - 2\left(\frac{R}{c}\right)\sqrt{\left(\frac{R}{c}\right)^2 - 1}$$

Here $R = 2+1.5 = 3.5\ cm$

$c = 1.5\ cm$. (Refer Table 6.1)

Therefore,

$$m = -1 + 2\left(\frac{3.5}{1.5}\right)^2 - 2\left(\frac{3.5}{1.5}\right)\sqrt{\left(\frac{3.5}{1.5}\right)^2 - 1}$$

$m = 0.050$

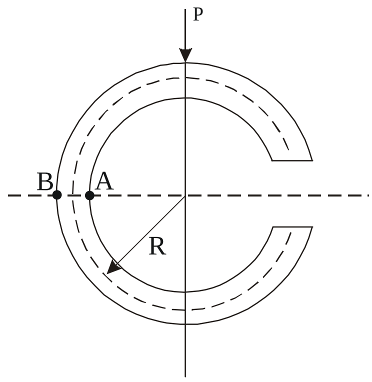


Figure 6.14 Loaded circular link

At section *AB*, the load is resolved into a load *P* and a bending couple whose moment is positive. The stress at *A* and *B* is considered to be the sum of the stress due to axial load *P*, and the stress due to the bending moment *M*.

Therefore, Stress at point *A* is

$$\sigma_{\theta A} = \sigma_A = \frac{P}{A} + \frac{M}{AR}\left[1 + \frac{y_A}{m(R + y_A)}\right]$$

$$= -\frac{1000}{7.06} + \frac{(3.5\times 1000)}{7.06\times 3.5}\left[1 + \frac{(-1.5)}{0.050(3.5 - 1.5)}\right]$$

or σ_A = -2124.65 *kg/cm*^{2} (compressive).

The stress at point *B* is given by

$$\sigma_{\theta_B} = \sigma_B = +\frac{P}{A} + \frac{M}{AR}\left[1 + \frac{y_B}{m(R + y_B}\right]$$

$$= \frac{-1000}{7.06} + \frac{3500}{7.06\times 3.5}\left[1 + \frac{1.5}{0.050(3.5 + 1.5)}\right]$$

$\therefore \sigma_B$ = 849.85 *kg/cm*^{2} (Tensile)

Comparison by Straight Beam Formula

The moment of inertia of the ring cross-section about the centroidal axis is

$$I = \frac{\pi d^4}{64} = \frac{\pi (3)^4}{64} = 3.976 cm^4$$

If the link is considered to be a straight beam, the corresponding values are

$$\sigma_A = \frac{P}{A} + \frac{My}{I}$$

$$= -\frac{1000}{7.06} + \frac{(+3500)(-1.5)}{3.976}$$

$\therefore \sigma_A$ = -1462.06 *kg/cm*^{2} (compressive)

& $\sigma_B = \frac{-1000}{7.06} + \frac{3500\times 1.5}{3.976}$

σ_B = 1178.8 *kg/cm*^{2} (tensile)

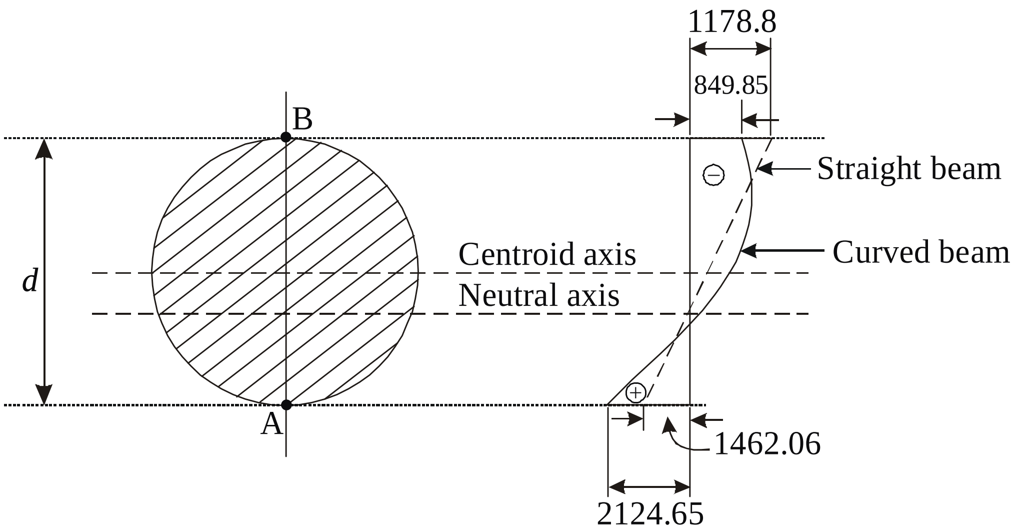


Figure 6.15 Stresses along the cross-section

Example 6.6

An open ring having *T*-Section as shown in the Figure 6.16 is subjected to a compressive load of 10,000 kg. Compute the stresses at *A* and *B* by curved beam formula.

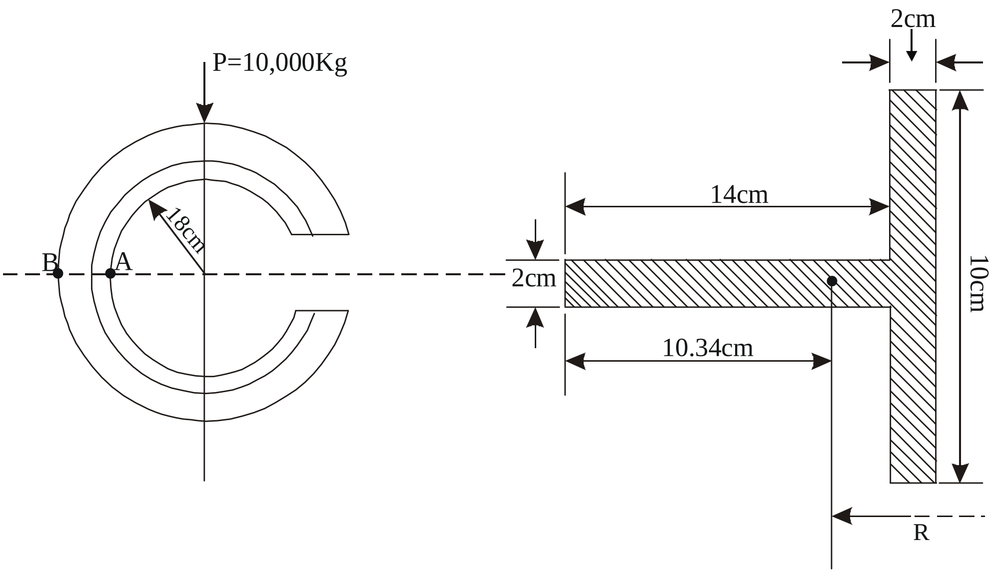


Figure 6.16 Loaded open ring

Solution:

Area of the Section = A = $2\times 10 + 2 \times 14 = 48$ cm^2

The value of m can be calculated from Table 6.1 by substituting $b_1 = 0$ for the unsymmetric *I*-section.

From Figure,

R = 18+5.66 = 23.66 cm

$c_1 = c_3$ = 10.34 cm

c_2 = 3.66 cm, c = 5.66 cm

t = 2 cm

b_1 = 0 , b = 10 cm

m is given by

$$m = -1 + \frac{R}{A}\left[b_1.\ln(R+c_1) + (t-b_1).\ln(R+c_3) + (b-t).\ln(R-c_2) - b.\ln(R-c)\right]$$

$$= -1 + \frac{23.66}{48}\left[0 + (2-0)\ln(23.66+10.34) + (10-2)\ln(23.66-3.66) - 10\ln(23.66-5.66)\right]$$

Therefore, m = 0.042

Now, stress at A,

$$\sigma_A = \frac{P}{A} + \frac{M}{AR}\left[1 + \frac{y_A}{m(R+y_A)}\right]$$

$$= -\frac{10000}{48} + \frac{(10000\text{x}23.66)}{48\text{x}23.66}\left[1 + \frac{(-5.66)}{0.042(23.66-5.66)}\right]$$

$\therefore$ σ_A = -1559.74 kg/cm^2 (compressive)

Similarly, Stress at B is given by

$$\sigma_B = \frac{P}{A} + \frac{M}{AR}\left[1 + \frac{y_B}{m(R+y_B)}\right]$$

$$= -\frac{10000}{48} + \frac{10000\times 23.66}{48\times 23.66}\left[1 + \frac{10.34}{0.042(23.66+10.34)}\right]$$

$\therefore$ σ_B = 1508.52 kg/cm^2 (tensile)

Example 6.7

A ring shown in the Figure 6.17, with a rectangular section is 4cm wide and 2cm thick. It is subjected to a load of 2,000 kg. Compute the stresses at A and B and at C and D by curved beam formula.

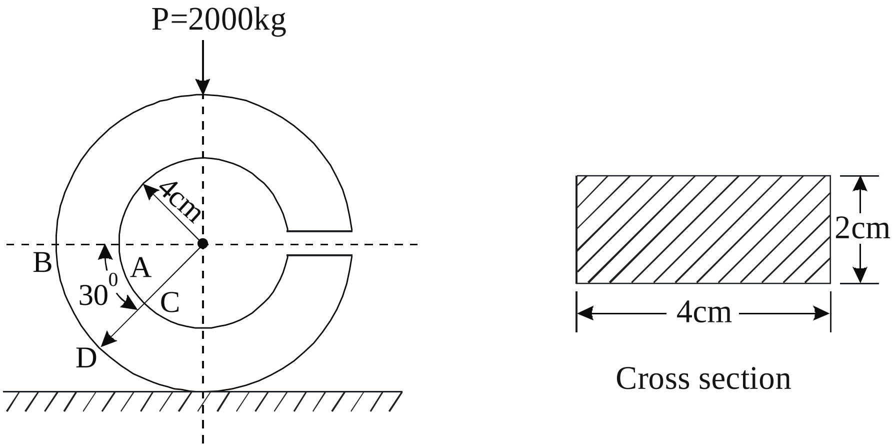


Figure 6.17 Loaded ring with rectangular cross-section

Solution**:** Area of the section $A = 4 \times 2 = 8cm^2$

The Radius of curvature of the centroidal axis = $R = 4 + 2 = 6cm.$

From Table 6.1, the m value for trapezoidal section is given by,

$$m = -1 + \frac{R}{Ah}\left\{\left[b_1 h + (R + c_1)(b - b_1)\right]\ln\left(\frac{R + c_1}{R - c}\right) - (b - b_1)h\right\}$$

But for rectangular section, $c = c_1, b = b_1,$

Therefore $m = -1 + \frac{R}{Ah}\left\{\left[bh + (R + c)(0)\right] \ln\left(\frac{R + c}{R - c}\right) - (0)\right\}$

$$m = -1 + \frac{6}{8 \times 4}\left\{\left[2 \times 4 + (6 + 2)(0)\right]\ln\left(\frac{6 + 2}{6 - 2}\right)\right\}$$

Therefore $m = 0.0397$

Now, stress at $A = \sigma_A = \frac{P}{A} + \frac{M}{AR}\left[1 + \frac{y_A}{m(R + y_A)}\right]$

$$= -\frac{2000}{8} + \frac{2000 \times 6}{8 \times 6}\left[1 + \frac{(-2)}{0.0397(6 - 2)}\right]$$

$$\therefore \sigma_A = -3148.6kg/cm^2 \text{ (Compression)}$$

Stress at $B = \sigma_B = \frac{P}{A} + \frac{M}{AR}\left[1 + \frac{y_B}{m(R + y_B)}\right]$

$$= \frac{-2000}{8} + \frac{2000 \times 6}{8 \times 6}\left[1 + \frac{2}{0.0397(6+2)}\right]$$

Therefore, $\sigma_B = +1574.31 kg/cm^2$ (Tension)

To compute the stresses at C and D

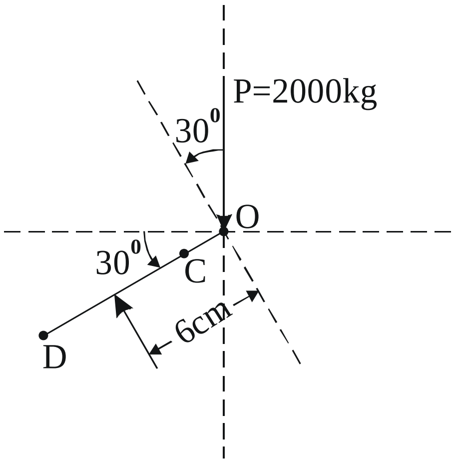


Figure 6.18

At section CD, the bending moment, $M = PR\cos 30^0$

i.e., $M = 2000 \times 6 \times \cos 30^0$

$= 10392\, kg - cm$

Component of P normal to CD is given by,

$N = P\cos 30^0 = 2000\cos 30^0 = 1732 kg.$

Therefore, stress at $C = \sigma_c = \frac{N}{A} + \frac{M}{AR}\left[1 + \frac{y_A}{m(R + y_A)}\right]$

$$= \frac{-1732}{8} + \frac{10392}{8 \times 6}\left[1 + \frac{(-2)}{0.0397(6-2)}\right]$$

$\therefore \sigma_c = -2726.7 kg/cm^2$ (Compression)

Stress at $D = \sigma_D = \frac{N}{A} + \frac{M}{AR}\left[1 + \frac{y_B}{m(R + y_B)}\right]$

$$= \frac{-1732}{8} + \frac{10392}{8 \times 6}\left[1 + \frac{2}{0.0397(6+2)}\right]$$

Therefore, $\sigma_D = 1363.4 kg/cm^2$ (Tension)

Example 6.8

The dimensions of a 10 tonne crane hook are shown in the Figure 6.19. Find the circumferential stresses σ_A *and* σ_B on the inside and outside fibers respectively at the section AB.

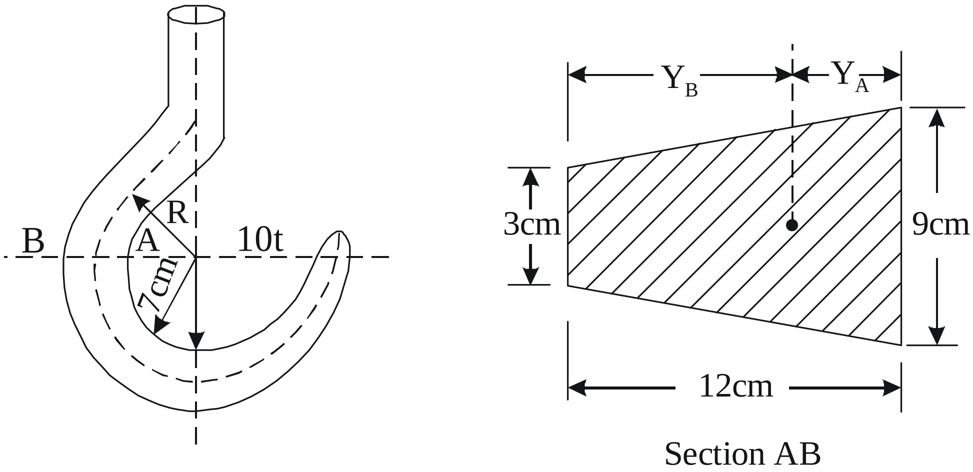


Figure 6.19 Loaded crane hook

***Solution*:** Area of the section $= A = \frac{9+3}{2} \times 12 = 72 cm^2$

Now, $y_A = \frac{12}{3}\left[\frac{9+2\times3}{9+3}\right] = 5cm.$

Therefore $y_B = (12-5) = 7cm.$

Radius of curvature of the centroidal axis $= R = 7 + 5 = 12cm.$

For Trapezoidal cross section, *m* is given by the Table 6.1 as,

$$m = -1 + \frac{12}{72\times12}\left\{[(3\times12)+(12+7)(9-3)].\ln\left(\frac{12+7}{12-5}\right) - (9-3)12\right\}$$

$$\therefore m = 0.080$$

Moment $= M = PR = 10{,}000 \times 12 = 120000\, kg - cm$

Now,

Stress at $A = \sigma_A = \frac{P}{A} + \frac{M}{AR}\left[1 + \frac{y_A}{m(R+y_A)}\right]$

$$= \frac{-10000}{72} + \frac{120000}{72 \times 12}\left[1 + \frac{(-5)}{0.08(12-5)}\right]$$

$$\therefore \sigma_A = -1240 kg / cm^2 \text{ (Compression)}$$

Stress at $B = \sigma_B = \dfrac{P}{A} + \dfrac{M}{AR}\left[1 + \dfrac{y_B}{m(R + y_B)}\right]$

$$= \frac{-10{,}000}{72} + \frac{120000}{72 \times 12}\left[1 + \frac{7}{0.08(12+7)}\right]$$

$$\therefore \sigma_B = 639.62 kg / cm^2 \text{ (Tension)}$$

Example 6.9

A circular open steel ring is subjected to a compressive force of 80 kN as shown in the Figure 6.20. The cross-section of the ring is made up of an unsymmetrical I-section with an inner radius of 150*mm*. Estimate the circumferential stresses developed at points A and B.

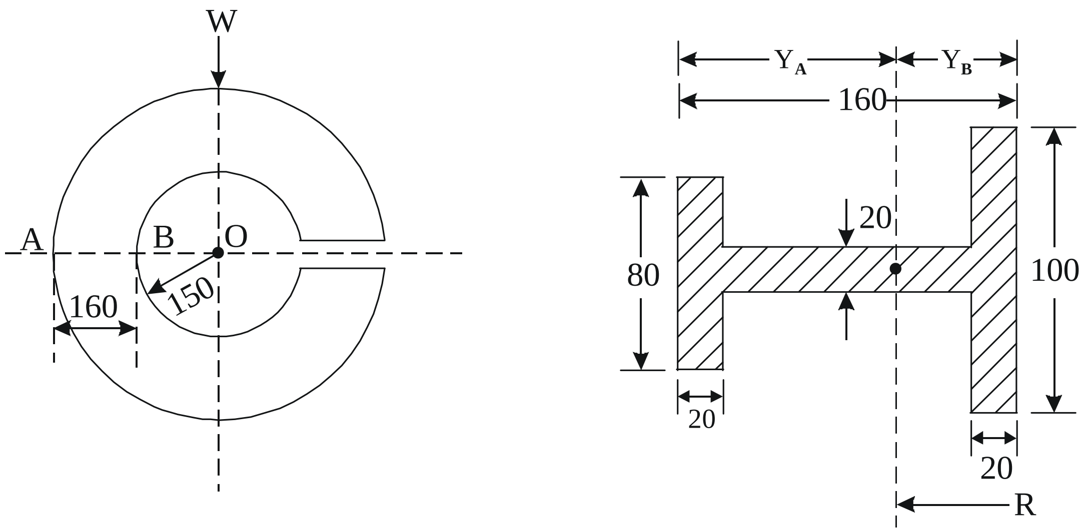


Figure 6.20 Loaded circular ring with unsymmetrical I-section

***Solution*:**

From the Table 6.1, the value of m for the above section is given by

$$m = -1 + \frac{R}{A}\left[b_1 \ln(R + c_1) + (t - b_1)\ln(R + c_3) + (b - t)\ln(R - c_2) - b\ln(R - c)\right]$$

Hence $R =$ Radius of curvature of the centroidal axis.

Now, $A = 20 \times 100 + 120 \times 20 + 80 \times 20 = 6000 mm^2$

$$y_B = \frac{(100 \times 20 \times 10) + (120 \times 20 \times 80) + (80 \times 20 \times 150)}{6000} = 75.33 mm.$$

$\therefore y_A = (160 - 75.33) = 84.67 mm.$

Also, $R = (150 + 75.33) = 225.33 mm.$

$$\therefore m = -1 + \frac{225.33}{6000}\begin{bmatrix} 80\ln(225.33 + 84.67) + (20 - 80)\ln(225.33 + 64.67) + \\ (100 - 20)\ln(225.33 - 55.33) - 100\ln(225.33 - 75.33) \end{bmatrix}$$

$\therefore m = 0.072.$

Moment $= M = PR = 80 \times 1000 \times 225.33 = 1.803 \times 10^7 \ N - mm.$

Now, Stress at point $B = \sigma_B = \frac{P}{A} + \frac{M}{AR}\left[1 + \frac{y_B}{m(R + y_B)}\right]$

$$\therefore \sigma_B = -\frac{80000}{6000} + \frac{1.803 \times 10^7}{6000 \times 225.33}\left[1 + \frac{(-75.33)}{0.072(225.33 - 75.33)}\right]$$

$\therefore \sigma_B = -93.02 \ N/mm^2$ (Compression)

Stress at point $A = \sigma_A = \frac{P}{A} + \frac{M}{AR}\left[1 + \frac{y_A}{m(R + y_A)}\right]$

$$= -\frac{80000}{6000} + \frac{1.803 \times 10^7}{6000 \times 225.33}\left[1 + \frac{84.67}{0.072(225.33 + 84.67)}\right]$$

$\therefore \sigma_A = 50.6 \ N/mm^2$ (Tension)

Hence, the resultant stresses at A and B are,

$\sigma_A = 50.6 \ N/mm^2$ (Tension), $\quad \sigma_B = -93.02 \ N/mm^2$ (Compression)

Example 6.10

Calculate the circumferential stress on inside and outside fibre of the ring at A and B, shown in Figure 6.21. The mean diameter of the ring is 5*cm* and cross-section is circular with 2*cm* diameter. Loading is within elastic limit.

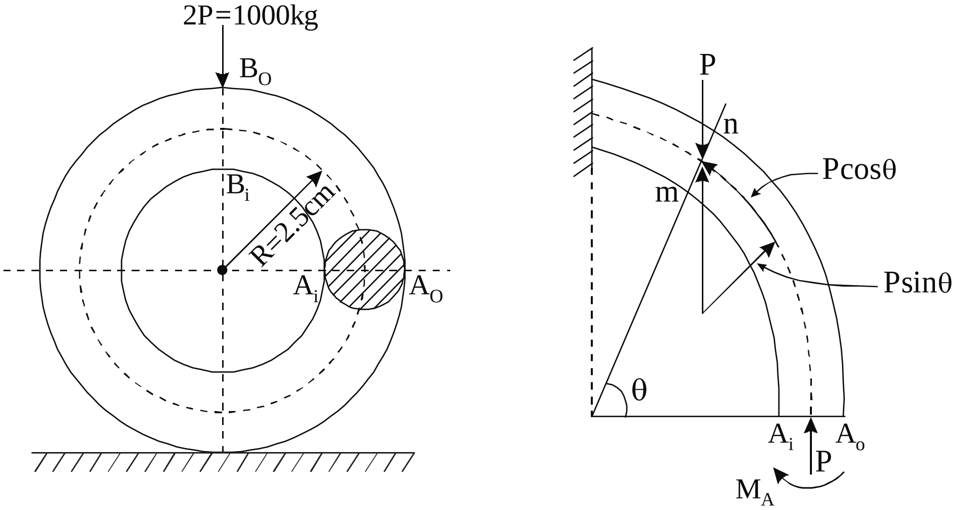


Figure 6.21 Loaded closed ring

***Solution*:** For circular section, from Table 6.1

$$m = -1 + 2\left(\frac{R}{c}\right)^2 - 2\left(\frac{R}{c}\right)\sqrt{\left(\frac{R}{c}\right)^2 - 1}$$

$$= -1 + 2\left(\frac{2.5}{1}\right)^2 - 2\left(\frac{2.5}{1}\right)\sqrt{\left(\frac{2.5}{1}\right)^2 - 1}$$

$$\therefore m = 0.0435$$

We have, $M_A = PR\left(1 - \frac{2}{\pi}\right)$

$$= 0.364PR = 0.364 \times 2.5P$$

$$\therefore M_A = 0.91P$$

Now, $\sigma_{A_i} = -\left(\frac{P}{A}\right) + \frac{M_A}{AR}\left[1 + \frac{y_i}{m(R + y_A)}\right]$

$$= -\left(\frac{P}{A}\right) + \frac{0.91P}{A \times 2.5}\left[1 + \frac{(-1)}{0.0435(2.5 - 1)}\right]$$

$$= -\left(\frac{P}{A}\right) - 5.21\left(\frac{P}{A}\right)$$

$$\therefore \sigma_{A_i} = -6.21\left(\frac{P}{A}\right) \text{ (Compressive)}$$

$$\sigma_{A_0} = -\left(\frac{P}{A}\right) + \frac{M}{AR}\left[1 + \frac{y_o}{m(R + y_B)}\right]$$

$$= -\left(\frac{P}{A}\right) + \frac{0.91P}{A \times 2.5}\left[1 + \frac{1}{0.0435(2.5 + 1)}\right]$$

$$\therefore \sigma_{A_o} = 1.755\left(\frac{P}{A}\right) \text{ (Tension)}$$

Similarly, $M_B = (M_A - PR)$

$$= (0.364PR - PR) = -0.636PR$$

$$= -0.636 \times 2.5P$$

$$\therefore M_B = -1.59P$$

Now, $\sigma_{Bi} = \frac{M_B}{AR}\left[1 + \frac{y_i}{m(R + y_i)}\right]$

$$= -\frac{1.59P}{A \times 2.5}\left[1 + \frac{(-1)}{0.0435(2.5 - 1)}\right]$$

$$\therefore \sigma_{Bi} = 9.11\left(\frac{P}{A}\right) \text{ (Tension)}$$

and $\sigma_{Bo} = -\left(\frac{1.59P}{A \times 2.5}\right)\left[1 + \frac{(+1)}{0.0435(2.5 + 1)}\right]$

$$= -4.81\left(\frac{P}{A}\right) \text{ (Compression)}$$

Now, substituting the values of $P = 500kg$,

$A = \pi(1)^2 = 3.14159\,cm^2$, above stresses can be calculated as below.

$$\sigma_{A_i} = -6.21 \times \frac{500}{\pi} = -988\,kg/cm^2$$

$$\sigma_{A_0} = 1.755 \times \frac{500}{\pi} = 279.32\,kg/cm^2$$

$$\sigma_{B_i} = 9.11 \times \frac{500}{\pi} = 1450\,kg/cm^2$$

$$\sigma_{B_0} = -4.81 \times \frac{500}{\pi} = -765.54\,kg/cm^2$$

Example 6.11

A ring of 200mm mean diameter has a rectangular cross-section with 50mm in the radial direction and 30mm perpendicular to the radial direction as shown in Figure 6.22. If the maximum tensile stress is limited to $120\,N/mm^2$, determine the tensile load that can be applied on the ring.

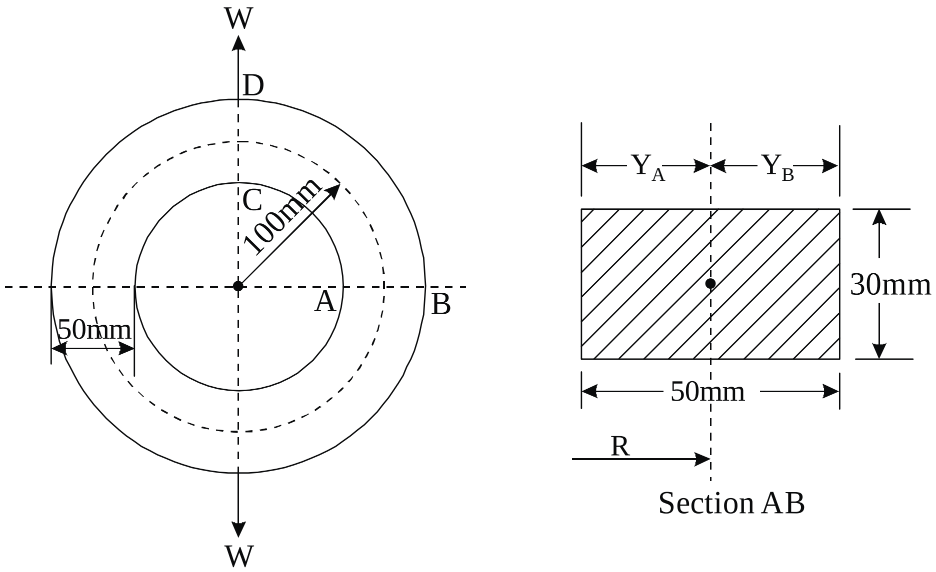


Figure 6.22 Closed ring with rectangular cross-section

***Solution*:** $R = 100mm$, Area of cross-section = $A = 30 \times 50 = 1500\,mm^2$

From Table 6.1, the value of m for the rectangular section is given by

$$m = -1 + \frac{100}{1500 \times 50}\left\{[30 \times 50 + 0]\ln\left(\frac{100+25}{100-25}\right) - 0\right\}$$

$$\therefore m = 0.0217$$

To find M_{AB}

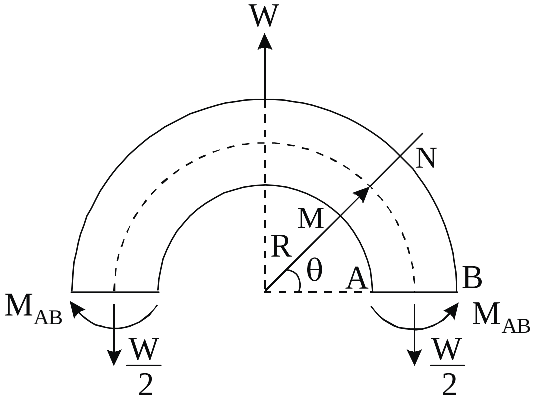


Figure 6.23

The Bending moment at any section MN can be determined by

$$M_{MN} = -M_{AB} + \frac{WR}{2}(1-\cos\theta)$$

$$\therefore At\,\theta = 0, \qquad M_{mn} = -M_{AB}$$

But $M_{AB} = \dfrac{WR}{2}\left(1-\dfrac{2}{\pi}\right)$

$$\therefore M_{AB} = \frac{W\times 100}{2}\left(1-\frac{2}{\pi}\right) = 18.17W$$

Now, $\sigma_A = \dfrac{P}{A} + \dfrac{M_A}{AR}\left[1+\dfrac{y_A}{m(R+y_A)}\right]$

$$= \frac{W}{2A} + \frac{M_A}{AR}\left[1+\frac{y_A}{m(R+y_A)}\right]$$

$$= \frac{W}{2\times 1500} + \frac{(-18.17W)}{1500\times 100}\left(1+\frac{(-25)}{0.0217(100-25)}\right)$$

$\therefore \sigma_A = 0.002073W$ (Tensile)

and $\sigma_B = \dfrac{P}{A} + \dfrac{M_A}{AR}\left[1+\dfrac{y_B}{m(R+y_B)}\right]$

$$= \frac{w}{2\times 1500} - \frac{18.17W}{1500\times 100}\left[1+\frac{25}{0.0217(100+25)}\right]$$

$\therefore \sigma_B = -0.00090423W$ (Compression)

To find stresses at C and D

We have, $M_{mn} = -M_{AB} + \frac{WR}{2}(1-\cos\theta)$

$\therefore At\,\theta = 90^0, \quad M_{mn} = M_{CD} = -M_{AB} + \frac{WR}{2}$

$\therefore M_{CD} = -18.17W + W \times \frac{100}{2} = 31.83W$

Now, stress at $C = \sigma_C = \frac{P}{A} + \frac{M_{CD}}{AR}\left[1 + \frac{y_C}{m(R+y_C)}\right]$

$$= 0 + \frac{31.83W}{1500\times100}\left[1 + \frac{(-25)}{0.0217(100-25)}\right]$$

$$= -0.00305W \text{ (Compression)}$$

and stress at $D = \sigma_D = \frac{P}{A} + \frac{M_{CD}}{AR}\left[1 + \frac{y_D}{m(R+y_D)}\right]$

$$= 0 + \frac{31.83W}{1500\times100}\left[1 + \frac{25}{0.0217(100+25)}\right]$$

$\therefore \sigma_D = 0.00217W$ (Tensile)

By comparison, the tensile stress is maximum at Point D.

$$\therefore 0.00217W = 120 \qquad \therefore W = 55299.54N \;\; or \;\; 55.3kN$$

Example 6.12

A ring of mean diameter 100mm is made of mild steel with 25mm diameter. The ring is subjected to four pulls in two directions at right angles to each other passing through the center of the ring. Determine the maximum value of the pulls if the tensile stress should not exceed $80\,N/mm^2$

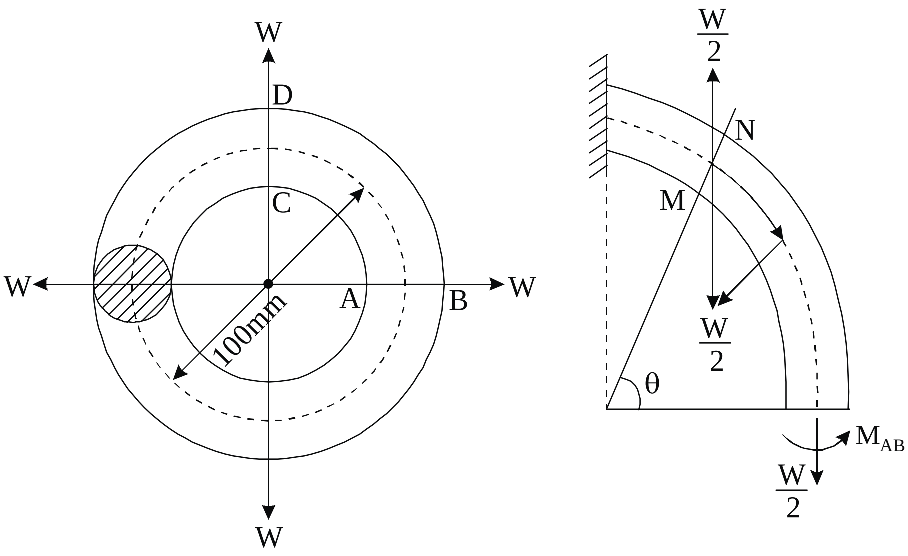


Figure 6.24 Closed ring with circular cross-section

***Solution*:** Here $R = 50mm$

From Table 6.1, the value of m for circular section is given by,

$$m = -1 + 2\left(\frac{R}{C}\right)^2 - 2\left(\frac{R}{C}\right)\sqrt{\left(\frac{R}{C}\right)^2 - 1}$$

$$= -1 + 2\left(\frac{50}{12.5}\right)^2 - 2\left(\frac{50}{12.5}\right)\sqrt{\left(\frac{50}{12.5}\right)^2 - 1}$$

$$\therefore m = 0.016$$

Area of cross-section = $A = \pi(12.5)^2 = 490.87\, mm^2$

We have, $M_A = \frac{WR}{2}\left(1 - \frac{2}{\pi}\right)$

$$= \frac{W}{2} \times 50\left(1 - \frac{2}{\pi}\right)$$

$$M_A = 9.085W$$

Now, $\sigma_A = \frac{P}{A} + \frac{M_A}{AR}\left[1 + \frac{y_A}{m(R + y_A)}\right]$

$$= \frac{W}{2 \times 490.87} - \frac{9.085W}{490.87 \times 50}\left[1 + \frac{(-12.5)}{0.016(50 - 12.5)}\right]$$

$= 0.0084W$ (Tensile)

$$\therefore \sigma_B = \frac{P}{A} + \left(\frac{M_A}{-AR}\right)\left[1 + \frac{y_B}{m(R + y_B)}\right]$$

$$= \frac{W}{2 \times 490.87} - \frac{9.085W}{490.87 \times 50}\left[1 + \frac{12.5}{0.016(50 + 12.5)}\right]$$

$\therefore \sigma_B = -0.00398W$ (Compression)

Also, $M_{CD} = (M_A - PR) = \left(-9.085 + \frac{W}{2} \times 50\right)$

$\therefore M_{CD} = +15.915W$

Now, $\sigma_C = \frac{M_{CD}}{AR}\left[1 + \frac{y_C}{m(R - y_C)}\right]$

$$= +\frac{15.918W}{490.87 \times 50}\left[1 + \frac{(-12.5)}{0.016(50 - 12.5)}\right]$$

$\therefore \sigma_C = -0.013W$ (Compression)

and $\sigma_D = +\frac{15.918W}{490.87 \times 50}\left[1 + \frac{12.5}{0.016(50 + 12.5)}\right]$

$= 0.0088W$ (Tension)

Stresses at Section CD due to horizontal Loads

We have, moment at any section MN is given by

$$M_{MN} = -M_A + \frac{PR}{2}(1 - \cos\theta)$$

At section CD, $\theta = 0$,

$$\therefore M_{CD} = -M_A + \frac{W}{2}R(1 - \cos 0)$$

$$M_{CD} = -M_A = -9.085W$$

$$\therefore \sigma_C = \frac{P}{A} + \frac{M_{CD}}{AR}\left[1 + \frac{y_C}{m(R + y_C)}\right]$$

$$= \frac{W}{2 \times 490.87} + \frac{(-9.085W)}{490.87 \times 50}\left[1 + \frac{(-12.5)}{0.016(50 - 12.5)}\right]$$

$\therefore \sigma_C = 0.00836W$ (Tensile)

and $\sigma_D = \frac{P}{A} + \frac{M_{CD}}{AR}\left[1 + \frac{y_D}{m(R + y_D)}\right]$

$$= \frac{W}{2 \times 490.87} + \frac{(-9.085W)}{490.87 \times 50}\left[1 + \frac{12.5}{0.016(50 + 12.5)}\right]$$

$\therefore \sigma_D = -0.00398W$ (Compression)

Resultant stresses are

$\sigma_C = (-0.013W + 0.00836W) = -0.00464W$ (Compression)

$\sigma_D = (0.0088W - 0.00398W) = 0.00482W$ (Tension)

In order to limit the tensile stress to $80\,N/mm^2$ in the ring, the maximum value of the force in the pulls is given by

$0.00482W = 80$

$\therefore W = 16597.51\,N$ *or* $16.598\,kN$

6.3.5 EXERCISES

1. Is the following function a stress function?

$$\phi = -\left(\frac{P}{\pi}\right) r\theta \sin\theta$$

If so, find the corresponding stress. What is the problem solved by this function?

2. Investigate what problem of plane stress is solved by the following stress functions

(a) $\phi = \frac{P}{K} r\theta \sin\theta$

(b) $\phi = -\frac{P}{\pi} r\theta \sin\theta$

3. Derive the equilibrium equation for a polar co-ordinate system.
4. Derive the expressions for strain components in polar co-ordinates.

5. Starting from the stress function $\phi = A\log r + Br^2 \log r + Cr^2 D$, obtain the stress components σ_r and σ_θ in a pipe subjected to internal pressure p_i and external pressure p_o. Obtain the maximum value of σ_θ when $p_o = 0$ and indicate where it occurs.
6. Check whether the following is a stress function
   $\phi = c\left[r^2(\alpha - \theta) + r^2 - r^2 \cos^2\theta \tan\alpha\right]$ where α is a constant.
7. Starting from the stress function $\phi = C_r + \frac{C_1}{r} - \frac{(3+\mu)}{8} + C_r^2 r^3$, derive expressions for σ_r and σ_θ in case of a rotating disk of inner radius 'a' and outer radius 'b'. Obtain the maximum values of σ_r and σ_θ.
8. Show that the stress function $\phi = A\log r + Br^2 \log r + Cr^2 + D$ solves the problem of axisymmetric stress distribution, obtain expressions for σ_r and σ_θ in case of a pipe subjected to internal pressure ρ_i and external pressure ρ_0.
9. Show that the following stress function solves the problem of axisymmetric stress distribution in polar coordinates
   $\phi = A\log r + Br^2 \log r + Cr^2 + D$
10. Explain axisymmetric problems with examples.
11. Derive the general expression for the stress function in the case of axisymmetric stress distribution.
12. Derive the expression for radial and tangential stress in a thick cylinder subjected to internal and external fluid pressure.
13. A curved bar bent into a arc of a circle having internal radius 'a' and external radius 'b' is subjected to a bending couple M at its end. Determine the stresses σ_r, σ_θ and $\tau_{r\theta}$.
14. For the stress function, $\phi = Ar^2 \log r$, where A is a constant, compute the stress components σ_r, σ_θ and $\tau_{r\theta}$.
15. A thick cylinder of inner radius 150*mm* and outer radius 200*mm* is subjected to an internal pressure of 15MN/m^2. Determine the radial and hoop stresses in the cylinder at inner and outer surfaces.
16. The internal and external diameters of a thick hollow cylinder are 80*mm* and 120*mm* respectively. It is subjected to an external pressure of 40MN/m^2, when the internal pressure is 120MN/m^2. Calculate the circumferential stresses at the external and internal surfaces and determine the radial and circumferential stresses at the mean radius.
17. A thick-wall cylinder is made of steel (E = 200*GPa* and $\nu = 0.29$), has an inside diameter of 20*mm*, and an outside diameter of 100*mm*. The cylinder is subjected to an

internal pressure of 300*MPa*. Determine the stress components σ_r and σ_θ at $r = a = 10mm$, $r = 25mm$ and $r = b = 50mm$.

18. A long closed cylinder has an internal radius of 100*mm* and an external radius of 250*mm*. It is subjected to an internal pressure of 80*MPa*. Determine the maximum radial, circumferential and axial stresses in the cylinder.
19. A solid disc of radius 200*mm* is rotating at a speed of 3000 *rpm*. Determine the radial and hoop stresses in the disc if $\nu = 0.3$ and ρ = 8000kg/m^3. Also determine the stresses in the disc if a hole of 30*mm* is bored at the centre of the disc.
20. A disc of 250*mm* diameter has a central hole of 50*mm* diameter and runs at 4000*rpm*. Calculate the hoop stresses. Take $\nu = 0.25$ and ρ = 7800 kg/m^3.
21. A turbine rotor 400*mm* external diameter and 200*mm* internal diameter revolves at 1000*rpm*. Find the maximum hoop and radial stresses assuming the rotor to be thin disc. Take the weight of the rotor as 7700 kg/m^3 and poisson's ratio 0.3.
22. Investigate what problem of plane stress is solved by the following stress function $\phi = \frac{3F}{4C}\left\{xy - \frac{xy^3}{3C^2}\right\} + \frac{P}{2}y^2$. Check whether the following is a stress function
$$\phi = \left(Ar^2 + Br^2 + \frac{C}{r^2} + D\right)\cos 2\theta$$
23. Show that $\left[Ae^{\alpha y} + Be^{-\alpha y} + Cye^{\alpha y} + Dye^{-\alpha y}\right]\sin \alpha x$ represents stress function.
24. The curved beam shown in figure has a circular cross-section 50mm in diameter. The inside diameter of the curved beam is 40mm. Determine the stress at B when $P = 20kN$.

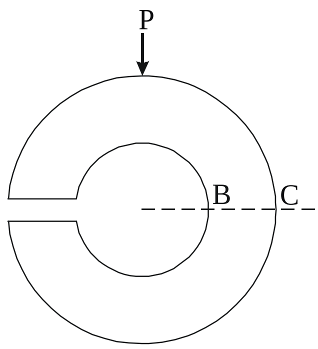


Figure 6.25

25. A crane hook carries a load $W = 20kN$ as shown in figure. The cross-section *mn* of the hook is trapezoidal as shown in the figure. Find the total stresses at points *m* and *n*. Use the data as given $b_1 = 40mm,\ \ b_2 = 10mm,\ \ a = 30mm$ and $c = 120mm$

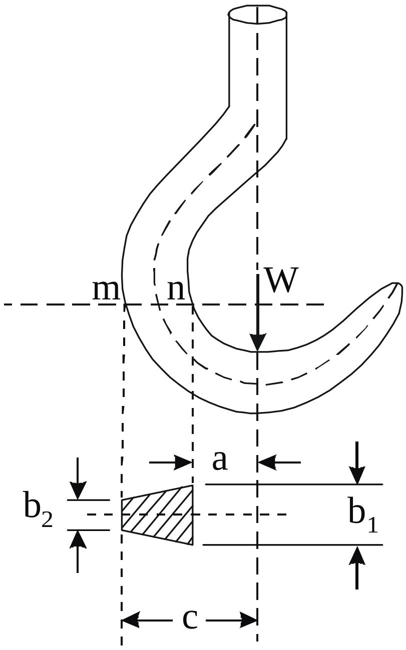


Figure 6.26

26. A semicircular curved bar is loaded as shown in figure and has a trapezoidal cross-section. Calculate the tensile stress at point A if $P = 5kN$

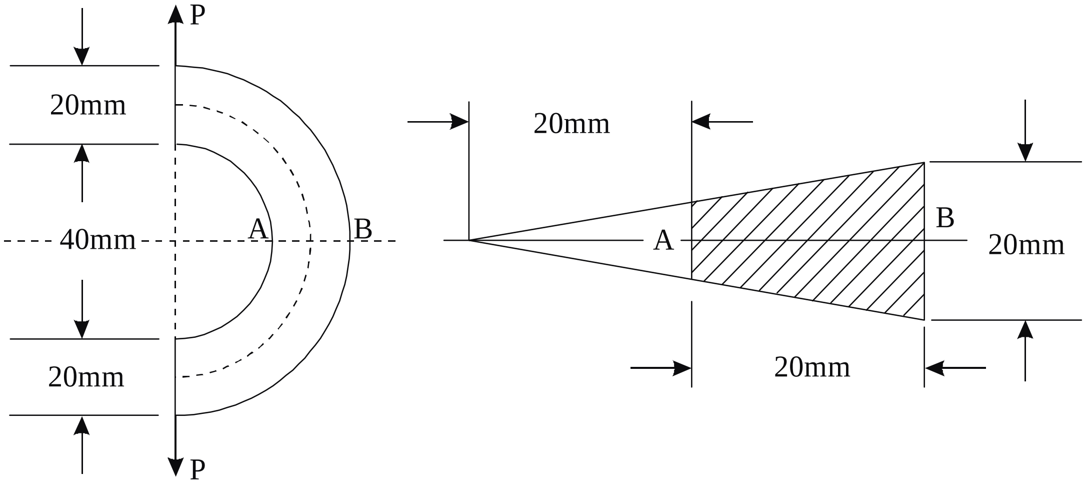


Figure 6.27

27. A curved beam with a circular centerline has a T-section shown in figure below. It is subjected to pure bending in its plane of symmetry. The radius of curvature of the concave face is 60mm. All dimensions of the cross-section are fixed as shown except the thickness *t* of the stem. Find the proper value of the stem thickness so that the extreme fiber stresses are bending will be numerically equal.

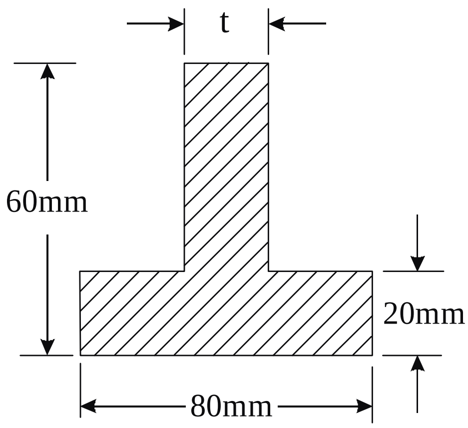


Figure 6.28

28. A closed ring of mean diameter 200mm has a rectangular section 50mm wide by a 30mm thick, is loaded as shown in the figure. Determine the circumferential stress on the inside and outside fiber of the ring at A and B. Assume $E = 210kN/mm^2$

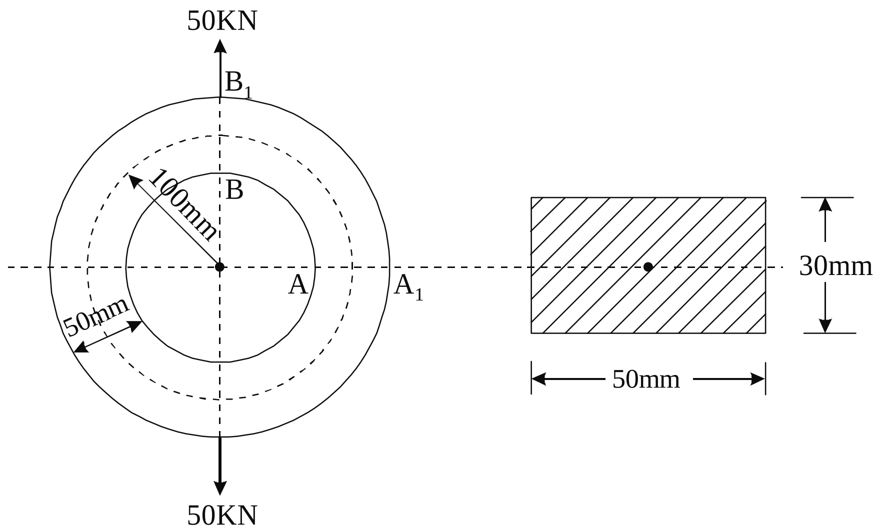


Figure 6.29

29. A hook has a triangular cross-section with the dimensions shown in figure below. The base of the triangle is on the inside of the hook. The load of $20kN$ applied along a line 50mm from the inner edge of the shank. Compute the stress at the inner and outer fibers.

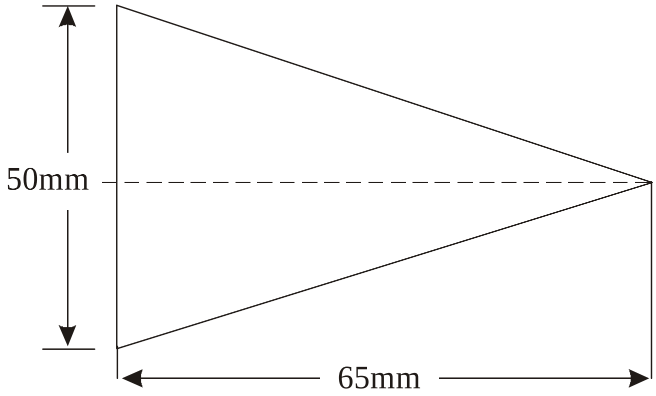


Figure 6.30

30. A circular ring of mean radius 40mm has a circular cross-section with a diameter of 25mm. The ring is subjected to diametrical compressive forces of 30*kN* along the vertical diameter. Calculate the stresses developed in the vertical section under the load and the horizontal section at right angles to the plane of loading.